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Propagation of sound in two species Bose-Einstein condensates

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We investigate the propagation of zero sound in two-species interpenetrating Bose-Einstein condensates. In very elongated clouds, this propagation is shown to be essentially one-dimensional. We also present 1D numerical experiments that exhibit the existence of two different sound pulses with velocities consistent with the theoretical predictions.

The dynamical behaviour of one-component Bose-Einstein condensates (BEC) has been the subject of numerous experimental and theoretical studies [1,2]. In particular, measurements of the velocity of sound pulses propagating in a cloud of ^{23}Na atoms have been shown to agree with the prediction of Bogoliubov theory [3]. On the other hand, Myatt *et al.* have reported the observation of a double condensate created by thermal contact between two different hyperfine states of ^{87}Rb [4]. This technique of sympathetic cooling combined with the design of an optical dipole trap [5] opens up the way to the realization of different multi-component condensates. Several theoretical works have already been devoted to the study of the ground state properties of two-species Bose-Einstein condensates (TBEC) [6] as well as to the determination of the frequency of the lowest collective excitations which correspond to global oscillations of the clouds [7–9].

In this paper, we study the propagation of zero sound in large two species condensates. Such systems exhibit at long wavelengths two phonon branches. As in the case of pure condensates [10,11], we show that in long cigar-shaped clouds, this propagation takes place essentially along the long axis. The sound velocities then take a simple form that lends itself easily to numerical and experimental verifications.

Ultracold and dilute TBEC are well described by coupled Gross-Pitaevskii (GP) equations for the macroscopic wavefunctions:

$$i\hbar\partial_t\Psi_1 = [-\frac{\hbar^2}{2m_1}\underline{\nabla}^2 + V_{e1} + U_{11}|\Psi_1|^2 + U_{12}|\Psi_2|^2]\Psi_1, \tag{1}$$

$$i\hbar\partial_t\Psi_2 = [-\frac{\hbar^2}{2m_2}\underline{\nabla}^2 + V_{e2} + U_{22}|\Psi_2|^2 + U_{12}|\Psi_1|^2]\Psi_2, \tag{2}$$

together with the normalization conditions

$$\int d\underline{r}|\Psi_i|^2 = N_i, \tag{3}$$

where the numbers of particles are conserved.

In these systems, the low energy binary collisions can be characterized by the s-wave scattering lengths a_{ij}. The interactions strengths are given by

$$U_{ij} = 2\pi\hbar^2 a_{ij}[\frac{1}{m_i} + \frac{1}{m_j}], \tag{4}$$

U_{ij} is positive when the interaction is repulsive and negative for attractive potentials. Few works have been devoted to the determination of the scattering lengths between unlike alcalis. In the following, we restrict ourselves to the case $U_{11} > U_{22} > 0$. The second terms in Eqs. (1) and (2) are the anisotropic harmonic confining potentials:

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$$V_{ei} = \frac{m_i \omega_{\perp i}^2}{2}[x^2 + y^2 + \lambda^2 z^2], \tag{5}$$

where $\frac{m_1\omega_{\perp 1}^2}{m_2\omega_{\perp 2}^2} = \frac{g_1}{g_2}$ [12]; g_i is the Landé g factor. We only consider mixtures of alkali atoms for which $g_1 = g_2$ and Eq. (5) then becomes

$$V_e(\underline{r}) = \frac{\kappa}{2}R^2; \qquad R^2 = x^2 + y^2 + \lambda^2 z^2, \tag{6}$$

where $\kappa = m_i\omega_{\perp i}^2$.

As in the MIT experiments [3], we investigate the case of very anisotropic cigar-shaped potentials for which the asymmetry parameter $\lambda = \frac{\omega_\perp}{\omega_{||}}$ is very small. The long axis lies in the z direction.

In order to discuss the behaviour of collective excitations, it is convenient to introduce the Madelung transformation:

$$\Psi_i = \sqrt{n_i}e^{i\phi_i}, \tag{7}$$

where n_i is the number density of the species i and, in absence of vortices, the phases ϕ_i play the role of potential for the two irrotational velocities $\underline{v}_i = \frac{\hbar}{m_i}\underline{\nabla}\phi_i$. In terms of these variables, the coupled GP equations take the following simple form:

$$\frac{\partial n_i}{\partial t} + \underline{\nabla}.(n_i\underline{v}_i) = 0, \tag{8}$$

$$m_i\frac{\partial \underline{v}_i}{\partial t} + \underline{\nabla}\{V_e(\underline{r}) + \sum_j U_{ij}n_j + \frac{m_i v_i^2}{2} + p_{qi}\} = 0, \tag{9}$$

where

$$p_{qi} = -\frac{\hbar^2}{2m_i\sqrt{n_i}}\underline{\nabla}^2(\sqrt{n_i}) \tag{10}$$

is the quantum pressure.

When the numbers of atoms are sufficiently large, the density profiles become smooth and the kinetic energy is small with respect to the interaction and potential energies. It only takes a significant contribution at the boundaries. When this term is neglected ($p_{qi} = 0$), one can obtain simple analytical expressions for the density distributions in the atomic clouds (Thomas-Fermi approximation); in the case where $U_{11} > U_{22}$ we get

$$n_{T2}(\underline{r}) = \frac{\kappa\delta U_{11}}{2\Delta}[R_2^2 - R^2]; \quad for\ R < R_2, \tag{11}$$

and similarly for n_1:

$$n_{T1}(\underline{r}) = \frac{\kappa}{2U_{11}}[R_1^2 - R_2^2] + \frac{\kappa\delta U_{22}}{2\Delta}[R_2^2 - R^2]; \quad for\ R < R_2, \tag{12}$$

and

$$n_{T1}(\underline{r}) = \frac{\kappa}{2U_{11}}[R_1^2 - R^2]; \quad for\ R_2 < R < R_1, \tag{13}$$

and the profiles vanish elsewhere.

In these equations, $\delta U_{ii} = U_{ii} - U_{12}$ and $\Delta = U_{11}U_{22} - U_{12}^2$. The radii R_1 and R_2 of the clouds in the transverse plane at $z = 0$ can be determined with the use of Eq. (3).

In free space ($V_e = 0$), the binary mixture is thermodynamically stable with respect to phase separation if

$$U_{11}U_{22} - U_{12}^2 > 0; \qquad \Delta > 0. \tag{14}$$

In the presence of the confining potential the two coupled clouds interpenetrate like miscible fluids as long as $\delta U_{22} > 0$ and the species with the largest self-interaction term (eg. 1) is more spread (cf. the bottom image in Fig. (1)). From Eq. (12) one can see that the species 1 experience an inverted harmonic potential near the origin ($R < R_2$) when $\delta U_{22} < 0$. The peak density of 1 does not coincide anymore with that of the other species, but it takes place at

$R = R_2$. If the self-interaction δU_{22} is further decreased the species 1 is expelled from the center of the trap and form a shell at the periphery [6,14]. In the following, we consider the case of two interpenetrating condensates, i.e. $\Delta > 0$ and $\delta U_{22} > 0$.

It is useful to first recall the main formulas describing the propagation of sound in absence of external potential. The densities n_{0i} of the uniform condensate are then equal to the peak densities at $\underline{r} = 0$ obtained from Eqs. (11) and (12). The frequencies ω_κ of the perturbations about this uniform condensate and proportional to $e^{i[\omega_\kappa t - \underline{\kappa}.\underline{r}]}$ are given by the following dispersion relations:

$$\omega_{\kappa\pm} = \frac{\kappa}{2}\{[\tilde{U}_{11} + \tilde{U}_{22}] \pm [(\tilde{U}_{11} - \tilde{U}_{22})^2 + 4\tilde{U}_{12}^2]^{1/2}\}^{1/2}, \tag{15}$$

where

$$\tilde{U}_{ii} = \frac{\hbar^2\kappa^2}{2m_i^2} + \frac{2U_{ii}n_{0i}}{m_i},$$

and

$$\tilde{U}_{12} = 2U_{12}\{\frac{n_{01}n_{02}}{m_1 m_2}\}^{1/2}.$$

Taking $U_{12} = 0$ and $U_{11} = 0$ or $U_{22} = 0$ in Eq. (15), one recovers the standard Bogoliubov dispersion relations for the pure condensates of each species. In a TBEC, the mode corresponding to composition inhomogeneities is propagative and not diffusive as in the case of normal binary mixtures. Both frequencies in Eq. (15) are gapless and in the long wavelength limit ($\kappa \to 0$), they yield two phonon-like branches:

$$\omega_\pm(\kappa \to 0) \to c_\pm \kappa, \tag{16}$$

where the corresponding sound velocities are given by [15]

$$c_\pm = \frac{1}{\sqrt{2}}\{({c_{01}}^2 + {c_{02}}^2) \pm [({c_{01}}^2 - {c_{02}}^2)^2 + \frac{4U_{12}^2 n_{01}n_{02}}{m_1 m_2}]^{1/2}\}^{1/2}, \tag{17}$$

where $c_{0i} = (\frac{U_{ii}n_{0i}}{m_i})^{1/2}$ is the Bogoliubov expression for the sound velocity of a pure condensate of the species i. These collisionless modes correspond to the Goldstone modes which proceed from the spontaneous breakdown of the gauge symmetries $\bar{\Psi}_i \to e^{i\phi_i}\Psi_i$ in the composite condensate. In the mode associated to the frequency ω_+, the two components interfere constructively whereas this interference is destructive in the case of ω_-. As a result, the corresponding sound velocity c_- can become imaginary signalling a long wavelength instability leading to phase separation when $\Delta < 0$ [16]. This condition is consistent with the thermodynamic criterion (cf. Eq. (14)).

In the presence of the confining potential, the linearized equation for the perturbations $\delta n_i(\underline{r}, t)$ about the stationary profiles n_{ei} take the following form after elimination of the velocity fields:

$$\frac{\partial^2 \delta n_1}{\partial t^2} = \underline{\nabla}.\{\frac{U_{11}n_{e1}}{m_1}\underline{\nabla}\delta n_1\} + \underline{\nabla}.\{\frac{U_{12}n_{e1}}{m_1}\underline{\nabla}\delta n_2\}, \tag{18}$$

$$\frac{\partial^2 \delta n_2}{\partial t^2} = \underline{\nabla}.\{\frac{U_{12}n_{e2}}{m_2}\underline{\nabla}\delta n_1\} + \underline{\nabla}.\{\frac{U_{22}n_{e2}}{m_2}\underline{\nabla}\delta n_2\}. \tag{19}$$

When the densities are sufficiently high, the profiles n_{ei} are well described by the Thomas-Fermi approximation (Eqs. (11)-(13)). This dynamics is greatly simplified in very elongated traps such as those used in the MIT experiments on pure BECs [3]. Since the transverse dimensions are smaller than the longitudinal ones, the density profiles across the trap quickly reach their equilibrium values. After this short transient, the propagation is essentially one-dimensional and it can be described by a local density perturbation that depends only on the longitudinal dimension $\delta n_i(z;t)$ [10,11]. Moreover, at the lowest order in the smallness parameter λ, one may neglect the variation of the equilibrium profiles n_{ei} in the z direction (cylindrical geomety). Eqs. (18) and (19) then become after integration over the cross section of the cloud of species 1 perpendicular to the direction of propagation:

$$\frac{\partial^2 \delta n_1}{\partial t^2} = \frac{U_{11} < n_{e1} >}{m_1}(\frac{\partial^2 \delta n_1}{\partial z^2}) + \frac{U_{12} < n_{e1} >}{m_1}(\frac{\partial^2 \delta n_2}{\partial z^2}), \tag{20}$$

$$\frac{\partial^2 \delta n_2}{\partial t^2} = \frac{U_{12} < n_{e2} >}{m_2}(\frac{\partial^2 \delta n_1}{\partial z^2}) + \frac{U_{22} < n_{e2} >}{m_2}(\frac{\partial^2 \delta n_2}{\partial z^2}), \tag{21}$$

where

$$n_{ei} = \frac{2}{R_i^2}\int_0^{R_1} r dr n_{ei}(\underline{r}). \tag{22}$$

As a result, sound propagation in such very anisotropic systems can be described by coupled one-dimensional wave equations. The corresponding sound velocities are still given by Eq. (17), but the densities of the uniform condensates are now replaced by the average densities as over the large transverse section of the cloud.

The effect of a density inhomogeneity on such two-species Bose-Einstein condensates can thus be captured by one-dimensional numerical experiments. The initial conditions were created by imaginary time integration of the coupled GP equations in the presence of an harmonic confining potential. Once the initial condition was obtained, we perturbated the condensate thus causing the appearance of inhomogeneities in the center of our mixtures. This perturbation was created by instantaneously adding to the initial external potential a Gaussian tip in the middle of the confining region. It mimics the laser beam that is focused in the experiments in the center of the trap to create a repulsive dipole force [3]. A variant of this experiment consists of creating the initial condensate with an external potential that includes the Gaussian barrier and eliminating it instantaneously at time $t = 0$. Both constructions give qualitatively the same results. We show one such integration on Fig. (1). The parameters were chosen to guarantee that the mixture is thermodynamically stable and the clouds interpenetrate. For these parameters, the Thomas-Fermi approximation permits to calculate the extension of the condensates which are 462 and 570 μm. The numerically created condensates have widths of approximately 450 and 550 μm, indicating a fairly good agreement between both aproaches. At $t = 0$ we have switched on the above mentioned Gaussian barrier to perturbate the system. This generates two pulses which propagate symmetrically outwards. In the beginning one sees only one hump (on each side of the mixture). However, as time increases, one can distinguish two humps one of which outdistances the other. The position of these density minima varies linearly with time and the speeds of propagation can thus be extracted from the simulations. They are of approximately 10.4 and 8.1 cm/s. When we calculate them by using the Thomas-Fermi peak densities into the expressions for the sound speed, we find 11.7 and 9.4 cm/s. Again there is a fairly good agreement between the integration of the coupled GP equations and Thomas-Fermi approximation results.

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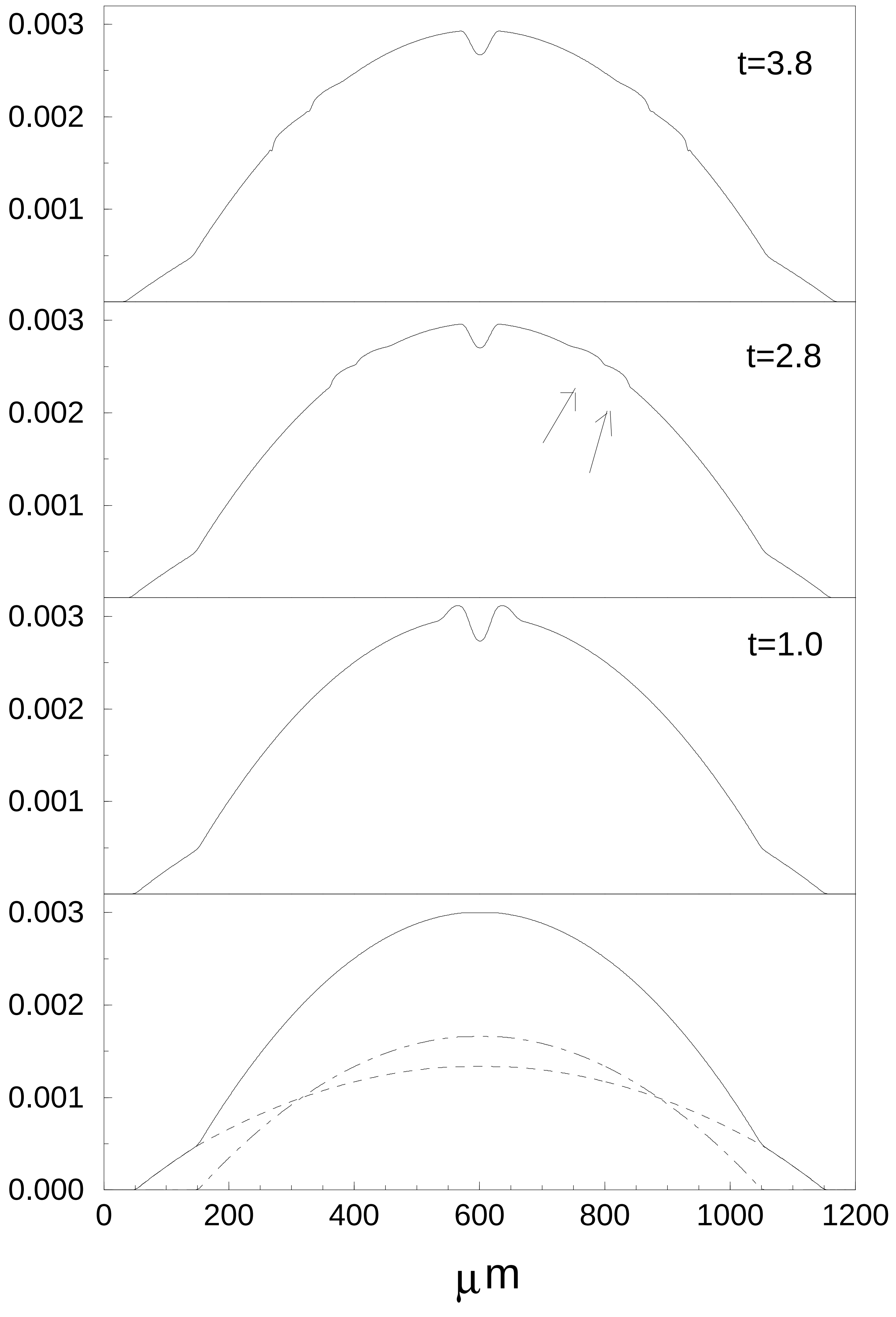
[1] F. Dalfovo *et al.*, cond-mat/9806038 (1998) and references therein.
[2] A. S. Parkins and D. F. Walls, Phys. Reports **303**, 4 (1998).
[3] M. R. Andrews *et al.*, Phys. Rev. Lett. **79**, 553 (1997); Phys. Rev. Lett. **80**, 2967 (1998).
[4] C. J. Myatt *et al.*, Phys. Rev. Lett. **78**, 586 (1997).
[5] D. M. Stamper-Kurn *et al.*, Phys. Rev. Lett. **80**, 2027 (1998).
[6] R. Ejnisman *et al.*, Optics Express **2**, 330 (1998) and references therein.
[7] Th. Busch *et al.*, Phys. Rev. A **56**, 2978 (1997).
[8] H. Pu and N. P. Bigelow, Phys. Rev. Lett. **80**, 1134 (1998).
[9] B. D. Esry and C. H. Greene, Phys. Rev. A **57**, 1265 (1998).
[10] E. Zaremba, Phys. Rev. A **57**, 518 (1998).
[11] G. M. Kavoulakis and C. J. Pethick, Phys. Rev. A **58**, 1563 (1998).
[12] T. L. Ho and V. B. Shenoy, Phys. Rev. Lett. **77**, 2595 (1996).
[13] D. M. Larsen, Ann. of Phys. **24**, 89 (1963).
[14] H. Pu and N. Bigelow, Phys. Rev. Lett. **80**, 1130 (1998).
[15] R. Graham and D. Walls, Phys. Rev. A **57**, 484 (1998).
[16] E. V. Goldstein and P. Meystre, Phys. Rev. A **55**, 2935 (1997).

Figure 1. Numerical integration of the coupled GP equations. The bottom image corresponds to the initial condition. The upper images show the development in time (increasing upwards and given in μs) after the perturbation has been applied. On the y-axis we plot the density of these condensates in μm^{-3}, while the abscissa represents their spatial extension in μm. The dashed line corresponds to condensate of particles 1. The dot-dashed line corresponds to condensate of particles 2. The sum of the densities is plotted as a solid line. For this figure the masses of both species are $10^{-26} Kg$. Each species contributes with 5 10^6 particles, and the frequency of the harmonic oscillator is equal to $300 Hz$. Particles of type 1 and 2 present s-wave scattering length of the 1.5 10^{-3} and 8.0 10^{-4} μm, while the interspecies length is of 1.0 10^{-4} μm. At $t = 0$ a Gaussian barrier was switched on to perturbate the mixture.


0.003
0.002
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t=3.8
0.003
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200
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